Errata in "Partial Identification Using Random Set Theory"*

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After our article Beresteanu, Molchanov and Molinari (*Journal of Econometrics* 166 (2012) 17–32, BMM henceforth) went into press, we found a *non sequitur* in the proof of Lemma B.2. Here we correct this lemma, and sharpen two results which use it. We also provide a list of typos that escaped us in the proof-reading stage.

Correction of Lemma B.2

Lemma B.2 Let X be a random compact set. Then a random vector x is stochastically smaller than X if and only if

$$(*) \qquad \mathbf{P}(x \in K_X) \geq \mathbf{P}\left(X \subset K_X\right),$$

for all sets K_X defined as $K_X = \bigcup_{\omega \in \Omega'} \{X(\omega) : X(\omega) \subset K\}$, where K is any compact set and Ω' is a fixed set of full probability.

Proof. Take a compact set K. By construction, $\mathbf{P}\left(X \subset K\right) = \mathbf{P}\left(X \subset K_X\right)$ and $K_X \subset K$. Hence, $\mathbf{P}(x \in K) \geq \mathbf{P}(x \in K_X)$ and if the dominance condition (2.2) in BMM holds for the set K_X, it also holds for K. ■

Remark. If X is a random compact interval on the line, the set K_X is necessarily a union of disjoint intervals. In this case, $\mathbf{P}\left(X \subset K_X\right)$ is the sum of the probabilities that X is a subset of each individual interval, and therefore it suffices to check condition (*) for K_X being any interval.

In light of the corrected Lemma B.2, the following amendments are provided:

- **Propositions 2.3 and 2.5:** $\tilde{K} = \tilde{K}(0) \cup \tilde{K}(1) \cup \cdots \cup \tilde{K}(T)$ (i.e., one should **not** take the convex hull of the set on the right hand side of the expression);
- **Proposition C.1:** The last sentence in the statement of the proposition needs to be deleted.

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Correction of Typos

- **Theorem 2.1**: In the statement of the theorem, "a random closed set X" should read "a random **compact** set X";

- **Proposition 2.6:** In the statement of the proposition, for the general case, $\mathrm{H}\left[\mathbf{P}(y(t))\right]$ should read:

$$\mathrm{H}\left[\mathbf{P}(y(t))\right] = \left\{\mu \in \Gamma_{\mathcal{Y}} : \mu\left(K\right) \geq \operatorname*{ess\,sup}_{v \in \mathcal{V}} \mathbf{P}\left(\overrightarrow{Y}\left(t\right) \subset K \middle| v\right) \;\; \forall K \in \mathcal{K}\left(\mathcal{Y}\right)\right\}.$$

  For $\mathcal{Y} = [0,1]$, $\mathrm{H}\left[\mathbf{P}(y(t))\right]$ should read

$$\begin{aligned}\mathrm{H}\left[\mathbf{P}(y(t))\right] \;\; = \;\; & \Big\{\mu \in \Gamma_{\mathcal{Y}} : \mu\left([k_1,k_2]\right) \geq \operatorname*{ess\,sup}_{v \in \mathcal{V}} \left[\mathbf{P}\left(y \leq k_2, z > t|v\right) 1\left(k_1 = 0\right) + \mathbf{P}\left(y \in \left[k_1,k_2\right], z = t|v\right)\right] \\ & + \mathbf{P}\left(y \geq k_1, z < t|v\right) 1\left(k_2 = 1\right)\Big] \;\; \forall k_1, k_2 \in \mathcal{Y} : k_1 \leq k_2\Big\}.\end{aligned}$$

  Similar corrections apply to the proof of this result.

- **Proof of Proposition 3.3:** Second column, line 11, $\mathbf{E}\left(\breve{w}\left(\psi - \breve{w}'\theta\right)\right)$ should be replaced by $\mathbf{E}\left(\breve{w}\left(\psi - \breve{w}'\breve{\theta}\right)\right)$.

- **Page 28:** Second column, line 25, "random closed set X" should read "random **compact** set X"

The authors apologize for the inconvenience caused by these errata.